November 28, 2017

1. (10 points) Imagine that \texttt{num\_orders} and \texttt{num\_instock} are particular values (as in a computer program). Write a negation for the following (Use De Morgan’s Law on the original statement form \((p \land q) \lor ((r \land s) \land t)):\)

\[
\begin{align*}
\text{\(num\_orders < 50\) and \(num\_instock > 300\))} \\
\text{or \((50 \leq num\_orders < 75\) and \(num\_instock > 500\))}
\end{align*}
\]

2. (10 points) Is the following a tautology? A contradiction? Neither?

\[
(\lnot p \lor q) \lor (p \land \lnot p).
\]

3. (10 points) Is \((p \oplus p) \lor r \equiv (p \land r) \oplus (p \land r)?\) Justify your answer (using a truth table).

4. (10 points) Show that \((p \rightarrow (q \rightarrow r)) \leftrightarrow ((p \land q) \rightarrow r).\) Alternatively, that \(p \rightarrow (q \rightarrow r) \equiv (p \land q) \rightarrow r.\)

5. (10 points) Write the following statements in logical form and show whether they are logically equivalent using a truth table and a short explanation.

- If \(2\) is a factor of \(n\) and \(3\) is a factor of \(n\), then \(6\) is a factor of \(n\).
- \(2\) is not a factor of \(n\) or \(3\) is not a factor of \(n\) or \(6\) is a factor of \(n\).

6. (10 points) Assume that the statement “If compound \(X\) is boiling, then its temperature must be at least 150\(^\circ\)C”. Which of the following statements is true? Justify your answers.

- (a) Compound \(X\) will boil only if its temperature is at least 150\(^\circ\)C.
- (b) If compound \(X\) is not boiling, then its temperature is at less than 150\(^\circ\)C.
- (c) A necessary condition for compound \(X\) to boil is that its temperature is at least 150\(^\circ\)C.
- (d) A sufficient condition for compound \(X\) to boil is that its temperature is at least 150\(^\circ\)C.
7. (20 points) You have an encounter with an island with knights (who always
    tell the truth) and knaves (who always lie). Two natives C and D approach
    and C says “Both of us are knaves.” What is C? What is D?

8. (20 points) You can assume the following statements:

(a) \( p \rightarrow q \)
(b) \( r \lor s \).
(c) \( \neg s \rightarrow \neg t \).
(d) \( \neg q \lor s \)
(e) \( \neg s \).
(f) \( (\neg p \land r) \rightarrow u \).
(g) \( w \lor t \).

Construct an argument for \( u \land w \). Do not use a truth table, instead justify
each conclusion you make using one of the argument forms introduced in
class (in Section 2.3 of book).

1 Suggested Problems

Problems in this section are not required for the homework. They are additional
problems that will help you in your progress. If you put forth a good faith effort
on all of these problems then you will receive 10 additional points on your
homework grade (not to exceed 100).

1. Is the following a tautology? A contradiction? Neither?
    \( (p \land p) \lor (\neg p \lor (p \land \neg q)) \).

2. Show that \( (p \lor q) \rightarrow r \equiv (p \rightarrow r) \land (q \rightarrow r) \).

3. Assume \( x \) represents a fixed real number. You may assume that \( (p \lor q) \rightarrow r \equiv (p \rightarrow r) \land (q \rightarrow r) \). Rewrite the following statement (using the above rule):
    If \( x > 2 \) or \( x < -2 \), then \( x^2 > 4 \).

4. Show that proof by division into cases is a valid argument form. That is, show
    \( ((p \lor q) \land (p \rightarrow r) \land (q \rightarrow r)) \rightarrow r \).

5. Give an example of a statement that is not logically equivalent to its
    converse.

6. Give an example of a statement that is not logically equivalent to its
    inverse.

7. Convert the following statement to if-then form: “passing the final exam
    is a necessary condition for passing the course.”